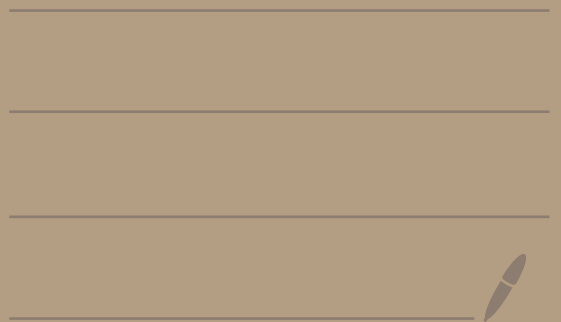


Math 4650

Test 2 Solutions



$$\textcircled{1} \sum_{n=1}^{\infty} \frac{1}{n^2+n}$$

$$\left. \begin{aligned} \frac{1}{n^2+n} &= \frac{1}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1} \\ 1 &= A(n+1) + Bn \\ \underline{n=-1:} \quad 1 &= B(-1) \rightarrow B = -1 \\ \underline{n=0:} \quad 1 &= A(1) \rightarrow A = 1 \end{aligned} \right\} \sum_{n=1}^{\infty} \frac{1}{n^2+n} = \sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1} \right)$$

partial sums:

$$s_1 = \frac{1}{1} - \frac{1}{2}$$

$$s_2 = \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) = 1 - \frac{1}{3}$$

$$s_3 = \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) = 1 - \frac{1}{4}$$

$\vdots$

In general,

$$s_k = 1 - \frac{1}{k+1}$$

$$\text{Thus,} \quad \sum_{n=1}^{\infty} \frac{1}{n^2+n} = \lim_{k \rightarrow \infty} s_k = \lim_{k \rightarrow \infty} \left(1 - \frac{1}{k+1}\right) = 1 - 0 = 1$$

(2)(a)

$$\begin{aligned}\sum_{n=3}^{\infty} \left(\frac{2}{3}\right)^n &= \left(\frac{2}{3}\right)^3 + \left(\frac{2}{3}\right)^4 + \left(\frac{2}{3}\right)^5 + \dots \\ &= \left(\frac{2}{3}\right)^3 \left[1 + \frac{2}{3} + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 + \dots \right] \\ &= \frac{8}{27} \left[\frac{1}{1 - 2/3} \right] = \frac{8}{27} [3] = \boxed{\frac{8}{9}}\end{aligned}$$

(2)(b)

$$\lim_{n \rightarrow \infty} \left| (-1)^n \frac{2n}{3n+2} \right| = \lim_{n \rightarrow \infty} \frac{2n}{3n+2} = \lim_{n \rightarrow \infty} \frac{2}{3 + \frac{2}{n}} = \frac{2}{3+0} = \frac{2}{3} \neq 0$$

Thus, $\lim_{n \rightarrow \infty} (-1)^n \frac{2n}{3n+2} \neq 0$

By the divergence test $\sum_{n=1}^{\infty} (-1)^n \frac{2n}{3n+2}$ diverges

(3)

Let $\varepsilon > 0$.

Note that

$$|x^2 - 4| = |x - 2| |x + 2|$$

Suppose $\delta \leq 1$.

If $0 < |x - 2| < \delta \leq 1$, then $-1 < x - 2 < 1$

which gives $3 < x + 2 < 5$

which gives $|x + 2| < 5$

Thus, if $|x - 2| < \delta \leq 1$, then

$$|x^2 - 4| = |x - 2| |x + 2| < |x - 2| \cdot 5$$

Let $\delta = \min \left\{ 1, \frac{\varepsilon}{5} \right\}$

Then $\delta \leq 1$ and $\delta \leq \frac{\varepsilon}{5}$.

So, if $0 < |x - 2| < \delta$, then

$$|x^2 - 4| < |x - 2| \cdot 5 < \frac{\varepsilon}{5} \cdot 5 = \varepsilon$$

$\delta \leq 1$

$\delta \leq \frac{\varepsilon}{5}$



(A) HW 5 #3

(B) HW 4 #3

(C) HW 5 #2

(D) HW 5 #1(c)