Math 4650

Test 2 Solutions

① $\sum_{n=1}^{\infty} \frac{1}{n^2+n}$

$$\frac{1}{n^2+n} = \frac{1}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1}$$

$$1 = A(n+1) + Bn$$

$\underline{n=-1:}$ $1 = B(-1) \to B = -1$

$\underline{n=0:}$ $1 = A(1) \longrightarrow A = 1$

$$\sum_{n=1}^{\infty} \frac{1}{n^2+n} = \sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1}\right)$$

partial sums:

$s_1 = \frac{1}{1} - \frac{1}{2}$

$s_2 = \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) = 1 - \frac{1}{3}$

$s_3 = \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) = 1 - \frac{1}{4}$

$\vdots \qquad \vdots$

In general,

$$s_k = 1 - \frac{1}{k+1}$$

Thus,

$$\sum_{n=1}^{\infty} \frac{1}{n^2+n} = \lim_{k\to\infty} s_k = \lim_{k\to\infty} \left(1 - \frac{1}{k+1}\right) = 1 - 0 = 1$$

2 (a)

$$\sum_{n=3}^{\infty}\left(\frac{2}{3}\right)^n = \left(\frac{2}{3}\right)^3 + \left(\frac{2}{3}\right)^4 + \left(\frac{2}{3}\right)^5 + \cdots$$

$$= \left(\frac{2}{3}\right)^3\left[1 + \frac{2}{3} + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 + \cdots\right]$$

$$= \frac{8}{27}\left[\frac{1}{1-2/3}\right] = \frac{8}{27}\left[3\right] = \boxed{\frac{8}{9}}$$

2 (b)

$$\lim_{n\to\infty}\left|(-1)^n\frac{2n}{3n+2}\right| = \lim_{n\to\infty}\frac{2n}{3n+2} = \lim_{n\to\infty}\frac{2}{3+\frac{2}{n}} = \frac{2}{3+0} = \frac{2}{3} \neq 0$$

Thus, $\lim_{n\to\infty}(-1)^n\frac{2n}{3n+2} \neq 0$

By the divergence test $\sum_{n=1}^{\infty}(-1)^n\frac{2n}{3n+2}$ diverges

(3)

Let $\varepsilon > 0$.

Note that

$$|x^2-4| = |x-2||x+2|$$

Suppose $\delta \le 1$.

If $0 < |x-2| < \delta \le 1$, then $-1 < x-2 < 1$

which gives $3 < x+2 < 5$

which gives $|x+2| < 5$

Thus, if $|x-2| < \delta \le 1$, then

$$|x^2-4| = |x-2||x+2| < |x-2| \cdot 5$$

Let $\delta = \min\{1, \frac{\varepsilon}{5}\}$

Then $\delta \le 1$ and $\delta \le \frac{\varepsilon}{5}$.

So, if $0 < |x-2| < \delta$, then

$$|x^2-4| \underbrace{<}_{\delta \le 1} |x-2| \cdot 5 \underbrace{<}_{\delta \le \frac{\varepsilon}{5}} \frac{\varepsilon}{5} \cdot 5 = \varepsilon$$

$\boxed{/\!/}$

(A) HW 5 # 3

(B) HW 4 # 3

(C) HW 5 # 2

(D) HW 5 # 1(c)